Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

On the Determination and Division of Surplus, and on the Modes of Returning it to the Contributors.

CONSIDERING the undeniable importance of a conformity to correct principles in carrying out the processes which must be resorted to for the purposes above mentioned, it is not a little remarkable that no work exclusively on the subject has hitherto appeared. If we except Mr. De Morgan's and Mr. Sang's Essays, and the observations scattered through the writings of Mr. Babbage, Mr. David Jones, Mr. Samuel Brown, and one or two others, we believe it may be truly said that scarcely any allusion to it is to be found.

We therefore propose, as many of our readers are interested in the discussion of such questions, and in the establishment of right methods in regard to them, to give some account of what appears to us to be the correct mode of proceeding. We have on former occasions endeavoured to establish some fundamental principles in connection with the subject, which we are satisfied cannot be controverted, and to these we shall still adhere. Further allusion to them now is needless, since they must necessarily be developed, and their accuracy more or less confirmed, as the several operations we propose to detail come to be described.

Let us suppose, then, that we are called upon, in the first place, to determine the present value of the risks of a Life Assurance Company, and thence the amount of surplus divisible amongst its members. Now it is evident that the payments required by the Company, or the premiums charged, form no guide to us in this inquiry; they are intended not merely to provide for the sum assured, they include in addition a provision against adverse fluctuations, and one to defray from year to year the expenditure of the establishment. These will clearly be required in every year to come as much as in those which have passed, and therefore must be carefully set aside for the purposes in question; but the portion of the premiums to be thus set aside must depend on that which is required to provide for the sum assured; so that the first consideration is to determine what part of the premium charged must be calculated upon as provision for the sum guaranteed under the policy. This again involves the further inquiry, What rate of mortality may be expected in future to prevail, and what rate of interest will actually be realized? It is evident that these questions can only be answered, with anything like an approximation to the truth, by a reference to the past, although there may exist circumstances which we have reason to know will influence the future, and render it prudent to modify accordingly whatever deductions we may be enabled to obtain from experience.

It will no doubt, however, constantly happen that the particular Society with which we have to do will not, either from its want of extent, or from its recent establishment, supply the required information; and in such cases we can only be guided by what has been observed in similar associations, making due allowance for any peculiarities in the one not possessed by the other. Thus, as regards mortality, we have the experience of the offices of the Equitable, and of the Amicable, all more or less confirmed by the Carlisle Table.

Some exercise of judgment will, no doubt, be required to determine which particular experience is best adapted to the case in point. It must, however, be remembered that there is no reason why corrections should not be applied at subsequent valuations. It is not to be expected that one unvarying rate will prevail throughout the existence of an association lasting, it may be, for centuries. If the variations occur, we must either persevere in an erroneous course, or apply such corrections so soon as we are satisfied of their necessity—in other words, we must judge on each occasion what experience most nearly resembles that we may look to have, and avail ourselves of it accordingly, taking especial care not to change our course till we are quite satisfied that there is good reason for it.

The rate of interest actually realized is not difficult to ascertain; a careful sifting of the accounts for the years which have succeeded the last valuation, will afford the means of determining it with tolerable accuracy. It will be needful, of course, to limit the inquiry to the rate of improvement of the premium fund, taking credit only for such interest as remains after other demands upon it have been satisfied. In this element, however, it is well known that much greater fluctuations are to be looked for than in that of mortality, and therefore it will be the more necessary to watch it carefully as the succeeding occasions arise, and to apply such corrections—in other words, to adopt such altered rate, as the circumstances may seem to require, or rather can be shown fully to warrant.

Having, then, determined on the true rates to be adopted in our calculations, and being provided with the usual tables of annual and single premiums, annuities, &c., answering to such rates, our next object will be to make a classification of all the assurances in accordance with the years in which the lives assured may be severally born. We are aware that it is a very common practice to estimate the value of each assurance separately, in which case such a classification is of course not needed. The only good reason for a separate estimate arises in the also common practice of dividing the surplus in proportion to the difference between the amount of premiums at compound interest and the value of each assurance; and this, it will be seen, renders the determination of the latter indispensable.

We believe, however, that we have clearly demonstrated, on a former occasion, that such a mode of division is entirely erroneous, and therefore there is no longer any corresponding advantage to compensate the excessive labour attendant upon a valuation of each risk separately; on the contrary, a very great disadvantage arises from the increased liability to error which such a proceeding necessarily involves. As regards any difference in the results of a separate and classed valuation, all experienced computers will at once admit that it is wholly insignificant. We believe we are quite within the mark in saying that the results would be identical even when the number of assurances in each class does not exceed six or eight. If the number in many of the classes be less than these, there is certainly nothing to be gained by a classification, but otherwise it is undeniably the preferable alternative.

In proceeding to make the required classification, it will be found most convenient to assume that all the lives were born on that day of the year at which the valuation is supposed to be made; and we would suggest also that the entries be regulated by the date of birth, and not by the *office* *age.** The lives in each class will thus, on the average, complete a given number of years on the day to which the accounts are made up. We are not ignorant that it is usual to adopt the office age, on the plea that the error, on the average, of half-a-year in each case caused by this arrangement, is in favour of the office. It is better, however, if avoidable, that there should be no error, that the truth should be rigorously adhered to throughout, and that any safeguards which may be required should be matter of distinct consideration.

As will appear hereafter, it is not necessary to make any distinction between participating and non-participating policies—the two may be blended together, provided only that they are for the whole term of life, and at a uniform premium: it may perhaps be desirable to affix some distinguishing mark against them. Of course any additions already made to the sums assured will have to be included, and due notice must be taken of any reduction in the original premium; and on this, as well as on other accounts, it is better to have fresh lists at each valuation, and not a species of debtor and creditor account in which the number of existing assurances is found after deducting the terminated ones. It is not necessary to take note of premiums being payable half-yearly or quarterly; the equivalent annual premium may safely be substituted in each case. To pursue a different course would render the classification of such assurances impracticable, whilst no advantage would accrue from the separate estimate of their value.

It may be well to mention here, too, that where an extra charge has been made for any deterioration in health, the corresponding age should always be considered as the true one, and the life classed accordingly. Where the extra charge is on account of residence abroad or for sea risk, it must be discarded, and treated separately. If the extra charge be one in accordance with tables specially made for any given climate, as is now generally the case with Indian risks, the assurance itself will come under a different class altogether, and must of course be omitted, to be valued eventually on the same principles as regards the foreign mortality as those adopted in reference to the native.

We have hitherto left it to be inferred that the premiums actually receivable by the Society are those to be entered in the lists; and for reasons which will appear, it is desirable that they should be, but it is not essential. The all-important point is the entry of a true premium, corresponding in each particular with the actual one. This operation may be rendered very simple by the previous preparation of a Table exhibiting the amount of premium at the true rate for £100, £200, £300, &c., so that mere inspection will enable anyone to make the entries from it. Omitting then such accessories as different computers may think it desirable to have, the lists will present the following appearance:—

---

* If the time of valuation be not *at* the 30th June, but the 30th Oct., for instance, then regard must be had to the financial, and not the civil year; so that one born any time in 1801, before the 30th April, must be considered as born in 1800—and so on.

Born in the Year			
Number of Policy.	Sum Assured, and Bonus.	Premium Receivable.	True Premium.
	£. s. d.	£. s. d.	£. s. d.

The entries being completed and the total of each of the last three columns carefully determined, the means are at hand for obtaining the required result. Let us call the total sum assured and additions S, the total premium receivable P, and the total true premium P′; and let us denote the value according to the true Table, of £1 payable at decease of any given life aged x years, by V_x, and the value of an annuity of £1 on such a life according to the same Table by A_x.

Then $S.V_x$=True Value of the sums assured and additions in each class,

$P' (A_x+.5)$=Value of true premiums,

$S.V_x—P' (A_x+.5)$=Liability,

$(P—P') (A_x+.5)$=Fund set apart for future surplus and expenses,

x in every case being equal to the difference between the year of valuation and that of birth. The addition is made of half unity to the value of the annuity in consideration of the premiums being one with another all due within six months from the time of the valuation; and although this may appear to be a somewhat loose approximation, we are satisfied it will be found abundantly near enough to the truth for all practical purposes. Indeed, as in the former case which we have alluded to, we believe the result, even though not more than half a dozen entries occur in the class, will be identical with that produced by a valuation of each Assurance in which the falling due of the premiums in different months of the year is taken into account.

If claims are payable sooner than at the expiration of six months from the day of death, the value of $S.V_x$ will have to be increased accordingly; but the interval of three months usually stipulated for, is generally much exceeded in practice.

We have said that although it was not essential to ascertain P, the amount of the annual premiums actually receivable, it was very desirable to have it. The only use hitherto made of it, as will be observed, has been to show the value of the marginal additions to the true premiums. It is useful, however, in some other respects. If the premium charged correspond with the true premium increased by a per centage, then $(P—P')\ (A_x+.5)$ ought to be that same per centage on $P'\ (A_x+.5)$, and thus a useful check is obtained. It may sometimes happen, too, when this per centage is unusually high, that it is considered proper to divide some portion of it, in order, it may be, to counterbalance any extraordinary fluctuation. In such a case it is only necessary to determine to what extent the future premiums shall remain "loaded," and the amount which may be anticipated is at once arrived at. Let us suppose, for instance, that $(P—P')\ (A_x+.5)$ is equal to 40 per cent. upon $P'\ (A_x+.5)$, and that it is thought advisable, under particular circumstances, to anticipate a portion of this fund—leaving a loading of not less than thirty per cent. to provide for future contingencies—here, then, we know immediately that one-fourth of $(P—P')\ (A_x+.5)$ may be divided, and that the prescribed provision for the future still remains untouched.

We should have mentioned that, as regards Assurances not susceptible of classification—such as those on an increasing scale, on two or more lives, or for terms—the same principles may be adhered to; that is to say, the several lives may all be considered as being born on the day of the year at which the valuation is made, and the annual premium assumed to be due at the end of six months from that time; for it will be obvious that the average will still obtain; and it must be remembered that the object in view is to determine the liability and surplus merely, and not the share of the several contributors in the last-mentioned quantity, that being ascertained by an entirely independent process.

Having, then, found the true value of the total sum assured and that of the premium payable in respect of it, the exact liability of the Company will be apparent; and this contrasted with the assets will, of course, indicate the amount of surplus to be divided.

In determining the value of the several items which constitute the assets, it is not desirable, in general, to make any alteration in the original cost of the security, for it must be remembered that the interest which they have yielded on the average *has been fixed upon as the true rate.* I need hardly mention that due allowance must be made where the annual proceeds are of a terminable character. It is by no means expedient to increase the value of a security which yields a high rate of interest; that circumstance implies that there are hazardous features in it, and the extra rate can only be properly looked upon as a provision against them, and not as constituting an increased value. In like manner due care must be taken that all property of a contingent nature be made absolute; an Assurance Company cannot treat assets of this character as a Reversionary one can—nor, indeed, would it purchase them on the same terms.

The surplus, then, being satisfactorily ascertained, it appears only consistent with justice and propriety that it or its value, as exactly as possible, should be returned to the contributors. The practice of reserving a part of it seems to be unnecessary, for there is not only the excess included in the future premiums, but very generally a subscribed capital to fall back upon, beside the fund reserved for the liability. The surplus is the property of the contributors of that day, and the complication of interests which must arise from the keeping back any part of it is carefully to be avoided. In short, the main object of the periodical adjustments made by Life Assurance Companies is, the return of what has been overpaid *to* *those who have contributed to the excess.* The living during the interval, terminated at a given valuation, are the persons naturally entitled to participate in the success or non-success of that particular interval; and the adjustment of their several interests being once made, ought not, on any account, to be subsequently disturbed. These are plain deductions from considering this system of Life Assurance as based upon an agreement that all the parties concerned shall be secured in the payment of a given sum at death, whenever that may happen, and *that the contributions required from them for that purpose shall be the least possible*—a system which places a well-defined and proper limit to the extreme aberrations commonly occurring in all cases of average, and which is probably as well arranged as the nature of the circumstances will admit.

Having, then, determined the amount to be divided amongst the members of the Society, we have next to consider what portion of it belongs to each of them, or, in other words, in what degree have they severally contributed towards it ? A little reflection will suffice to show that the surplus has arisen from the payments of that part of the premium charged which is in excess of the true premium, *for the amount of this last at the true rate of interest will necessarily constitute the liability at any given time.* Supposing, then, no expenses to have been incurred, the surplus will be exactly equal to the above-mentioned payments improved at the true rate of interest, and all that we have to do is to parcel out the total surplus in the proportion in which each member has thus contributed to form it; but expenses there must, of course, always be, and the real surplus will in consequence be much less than the supposititious one. Under such circumstances the nearest approach we can make to the truth will be the diminishing of each share of the surplus in the proportion which the actual surplus bears to the supposititious one, or, in other words, charging each of the contributors with a proportionate share of the total expenses. This is liable to some objection if all have not been members for the same period; but on the whole it appears, as we have elsewhere shown, to adjust the interests of each as fairly as is practicable. Probably the most convenient arrangement for effecting the required calculations is to have a volume of the proper thickness, one page being devoted to each Assurance, and the details being somewhat after the following fashion:—

	Years.	Months.	
No...... Sum assured, and Bonus			£. ,, ,, ,,
Actual Annual Premium			£. ,, ,, ,,
True ditto ditto			£. ,, ,, ,,
Difference (ϕ)			£. ,, ,, ,,
Logarithm of ditto			. . . . . .
Date of valuation	—	—	
Ditto of first payment of Premium since previous valuation	—	—	
Log. amount of £1 per annum in	—	—	. . . . .
Log. supposititious share of surplus (ϕa^n)			. . . . . .
Log. fraction $\frac{S}{\int\cdot\phi a^n}$			. . . . . .
Log. real share ditto	£. ,, ,, ,,		. . . . . .

the remaining space in the page being left for the calculation of the equivalent reversion, reduction in premium, &c. &c., to be hereafter described. By S in the foregoing arrangement is meant the actual surplus, and by $\int.\phi a^n$ the amount of the supposititious one, formed from the payments of ϕ. (that is the difference between the premium charged and the true premium) improved at the true rate of interest. The share of each member is thus found from the ratio:—

$\int.\phi a^n : S :: \phi a^n : \frac{S}{\int.\phi a^n} \times \phi a^n$. and as the fraction here is a constant quantity and a Table can be easily formed of the logarithms of the amount of £1 per annum paid in advance, in the number of months (usually not more than 60) entering into the calculation, the operation is very readily performed by the simple addition of the logarithms of the several quantities in question.

It will be obvious that before the value of the real share of surplus can be obtained the total amount of the supposititious shares must be determined, and for this purpose the results found in the foregoing statement will have to be tabulated after the following manner as the work proceeds:—

Number of Policy.	Suppos. Share. (ϕa^n)	Real Share. $\left(\frac{S}{\int.\phi a^n} \times \phi a^n\right)$			

the second column being completed before the third can be commenced. The remaining vacant columns will serve for the results connected with the allotment of the shares to the assured, the details of which we propose to discuss in our next number.